# A PRELIMINARY ASSESSMENT OF THE TITAN PLANETARY BOUNDARY LAYER

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## ABSTRACT

Preliminary estimates of the characteristic features of the Titan planetary boundary layer (PBL) are derived from the combined application of a patched Ekman–surface layer model and Rossby number similarity theory. A characteristic Ekman depth of ~0.7 km is anticipated, with an eddy viscosity $K \sim 10^3$ $cm^2 \cdot s^{-1}$, an associated friction velocity $u_* \sim 10^{-2}$ $m \cdot s^{-1}$, and a surface wind typically smaller than 0.6 $m \cdot s^{-1}$. Actual values of these parameters probably vary by as much as a factor of two or three, however, in response to local-temporal variations in surface roughness and stability. The saltation threshold for the wind-blown injection of ~50μm particulates into the atmosphere is less than twice the nominal friction velocity, suggesting that dusty breezes might be an occassional feature of the Titan meteorology. The direct measurement of Titan's PBL may be realized during the last two minutes of the Huygens Probe descent to its surface.

Keywords: Titan meteorology, boundary layer, surface wind, atmospheric circulation.

## 1. INTRODUCTION

The planetary boundary layer (PBL), is the region of turbulent eddy mixing immediately above the solid surface assoicated with the combined effects of wind shear and convection. It represents an important component of a planet's general circulation, mediating the transfer of angular momentum and heat between the surface and the atmosphere aloft. Successful models and theories of a planet's dynamic meteorolgy therefore depend to some extent upon the fidelity of their parameterization of the resulting surface drag. The PBL may also play a significant role in geological (and sea-surface?) processes by its effect on erosion, particulate transport, ground hydrology, and possible moist convection, for example. Important tests and constraints of these considerations will be provided by the *in situ* measurements of the Titan PBL by the Huygens Probe, as well as the anticipated characterization of its surface by the Cassini Orbiter RADAR.

## 2. *K* THEORY AND THE EKMAN-TAYLOR MODEL

A useful framework for the analysis of boundary layer flows is the first-order closure model relating the Reynolds-stress components of the correlated turbulent velocity fluctuations, $\tau_x = -\rho\langle u'w'\rangle$ and $\tau_y = -\rho\langle v'w'\rangle$, to the vertical mean flow shear in terms of an eddy viscosity $K$, with $\tau = \rho K \partial U/\partial z$ (e.g. Refs. 1, 2). In terms of this ($K$ theory) model, the horizontal vector momentum balance for the steady, eddy-viscous control of the geostrophic wind reads

$$f\hat{\mathbf{z}} \times (\mathbf{U} - \mathbf{G}) = \frac{\partial}{\partial z} K \frac{\partial \mathbf{U}}{\partial z}. \qquad (1)$$

Here $\mathbf{U}$ denotes the horizontal velocity vector, $\mathbf{G}$ the surface gradient wind vector (as determined by the horizontal gradient of surface pressure), $\hat{\mathbf{z}}$ the vertical unit vector, and $f \equiv 2\Omega\sin\lambda$ is the Coriolis parameter, with $\Omega$ the planetary rotation frequency and $\lambda$ the latitude.

The appropriate non-dimensional parameter for the geostrophic boundary layer regime is the Ekman number $E \equiv 2K/fD^2$ representing the ratio of the eddy viscous to Coriolis forces. For a fixed value of $K$, $E = 1$ for a characteristic depth $D = \delta_E \equiv (2K/f)^{1/2}$. This is the e-folding scale for the familiar constant-$K$ model of the Ekman wind spiral (Ref. 1). This prescribes the clockwise rotation of the wind vector by 45° between the surface (in the planetary hemisphere of counter-clockwise spin), assuming a vanishing velocity there, and an altitude $D_E = \pi(2K/f)^{1/2}$, where it matches the direction and asymptotically approaches the strength of the gradient flow aloft. $D_E$ therefore serves as a diagnostic measure of the total depth of the PBL. The "friction velocity" $u_* \equiv (|\tau_s|/\rho)^{1/2}$, characterizing the strength of the surface wind stress $|\tau_s|$, may be estimated as $U^* \sim 4f \cdot D_E$, according to the similarity theory discussed below.

For the Earth (Ref. 3) and Mars (Refs. 4,5), where the Ekman wind spiral may be inferred from actual measurements near the surface, the Ekman depth provides an estimate of the size of the eddy viscosity $K$. Alternatively, an independent estimate of $K$ provides a diagnostic characterization of the boundary layer depth and friction velocity. The intrepretive analysis of Voyager infrared measurements of Titan by Flasar *et al.* (Ref. 6) provides an upper limiting estimate of $K \le 10^3$ $cm^2 \cdot s^{-1}$ near the surface, based upon a consideration of the constraints imposed by the globally averaged vertical transport of angular momentum. The diagnostic interpretations of this estimate for Titan, in comparison with those for the other minor planet atmospheres of the Solar System (Refs. 3–8), are summarized in Table 1. In reality, $K$ may vary by a factor of ten or so in response to diurnally variable surface heating as well as local-temporal variations in the gradient wind. On Mars, for example, the friction velocity may approach values nearly twice that given in Table 1 during the extreme turbulent conditions associated with global dust storms (Ref.4). Although the estimated value of $K$ for Titan is more than an order of magnitude smaller than that for

Table 1: Eddy Viscosity Estimates for Minor Planet Atmospheres

| | **Eddy Viscosity** $K$ ($cm^2s^{-1}$) | [Ref.] | **PBL Parameters ($K$ Theory)** Ekman Depth $D_E=\pi(2K/\Omega)^{1/2}$ (km) | Friction Velocity $U_* \sim ?\ 4\Omega D_E$ ($m{\bullet}s^{-1}$) |
|---|---|---|---|---|
| Earth | $5 \times 10^4$ | [3] | 1 | 0.3 |
| Mars | $1\text{–}5 \times 10^5$ | [4,5] | 2–4 | 0.5–1 |
| Venus | $\sim 2 \times 10^4$? | [7] | 11? | 0.01? |
| Triton | $\sim 10^5$ | [8] | ~4 | ~0.2 |
| Titan | $\leq 10^3$ | [6] | 0.7 | 0.01 |

any other planetary atmosphere, the indicated depth of the PBL is comparable to that for the Earth, owing to its relatively slow planetary rotation, with $\Omega = 4.56 \times 10^{-6}\ s^{-1}$.

Another derived parameter of interest is the "spin-down time" characterizing the decay of vorticity within the effectively inviscid atmosphere aloft by Ekman pumping at the surface (Refs.1,3), estimated as $\tau_E = H(2/fK)^{1/2}$, where H is the pressure scale height. With $H \approx 20$km and $K \approx 10^3\ cm^2{\bullet}s^{-1}$ for Titan, $\tau_E \approx$ 500 (24 hour) days, as compared with about 4 days on the Earth.

Before proceeding with the further analysis of the Titan PBL based on solutions to Eq. 1, it is worth checking its validity as the appropriate governing balance for the eddy viscous control of horiontal motions for which the surface gradient flow is assumed to be essentially geostrophic. This demands that the synoptic-scale Rossby number $G/\Omega L << 1$ (Ref.1), where L is the characteristic horizontal length scale of the motion, comparable to the planetary radius $a$ for global-scale motions. The gradient thermal wind analysis of Voyager infrared measurements of an equator-to-pole contrast on Titan of about 20K near the 0.5mb level indicates a cyclostrophic, super-rotational flow of ~100 $m{\bullet}s^{-1}$ at stratospheric levels, for which $G/\Omega a \sim 8$ (Ref. 6). At near-surface levels, however, the Voyager 530 $cm^{-1}$ data suggest an equator–to–pole contrast of no more than about 3K, and possibly much smaller, owing to the convolved effects of stratospheric hazes in the interpretation of the infrared emission in this spectral region (Ref.9). Geostrophic scaling anticipates that the strength of the gradient wind near the top of the Titan PBL may be estimated as $G \sim g(\Delta T/T)D_E/\Omega a \approx 2.5\ m{\bullet}s^{-1}$ (for $\Delta T \approx 3K$, $T \cong 95K$, and a graviational acceleration $g = 1.35\ m{\bullet}s^{-2}$), so that $G << \Omega a \approx 12 m{\bullet}s^{-1}$ as required. For comparison, the application of Stone's radiative-dynamical model for the baroclinic adjustment of global tropospheric structure (Ref.10) to Titan, with a radiative cooling time of 95 years and a radiative equilibrium stability in the range –(0.1–2.0) $K{\bullet}km^{-1}$, as appropriate for a weak tropospheric greenhouse, predicts a mean geostrophic flow of about 1 $m{\bullet}s^{-1}$. These estimates suggest the presence of a geostrophic sub-layer within Titan's troposphere, underlying the cyclostrophic region above, for which the Ekman layer solution may indeed serve as a useful diagnostic model of the PBL structure. A *post facto* corroboration of this assumption will be derived below in consideration of eddy viscous speed limits on the surface flow. The application of the Ekman solution to Venus, where $\Omega a \approx 2\ m{\bullet}s^{-1}$, may be more problematic, although the estimated friction velocity indicated in Table 1 is of the same order of magnitude as that derived from anemometer measurements on board the Venera 9 surface lander (Ref. 11).

In the layer immediately adjacent to the surface, mixing length theory and observations suggest that the eddy viscosity varies with the elevation. In the limit $D \rightarrow 0$, the Ekman number $E \rightarrow \infty$, for which Eq. 1 reduces to the statement of constant shear stress. Assuming that for neutrally stable conditions U is a function only of the elevation z, with a no-slip (U=0) condition at the z = 0 level, this implies the logarithmic profile $U_s = k^{-1}u_* ln(1 + z/z_o)$, with $K = ku_*(z + z_o)$, where $k \cong 0.35$ is von Karmen's constant and $z_o$ is the roughness parameter, typically 1/30 the standard deviation in the surface topography. Typical terrestrial values for $z_o$ are ~1cm for level plains, ~1m for mountainous regions or large cities, and ~$10^{-4}$ m over smooth open seas.

The constant-stress layer obviously cannot extend to a large fraction of the Ekman scale height (where the stress is reduced by a factor ~1/e). One approach to the coherent model description of the PBL is an an inner (constant stress, $E \rightarrow \infty$) surface layer solution of height h patched to an outer (constant $K$, $E \rightarrow$ order 1) Ekman layer. Using complex variable notation (with $i \equiv \sqrt{-1}$) to denote the separate wind components u and v, parallel and perpendicular to the direction of the gradient wind G, the combined two-layer solution to Eq. 1 then reads

$$u + iv = \begin{cases} G[1 - \sqrt{2}\sin\alpha\, e^{-(1+i)(z-h)/\delta_E + i(\alpha-\pi/4)}] & \text{for } z>h \\ (u_*/k)\,[ln(1 + h/z_o)]e^{i\alpha} & \text{for } z<h, \end{cases} \quad (2)$$

where $\alpha$ is the surface cross-isobar angle and again $\delta_E \equiv (2K/f)^{1/2}$. Upon requiring the continuity of velocity at z = h,

$$ln(1 + \frac{h}{z_o}) = \frac{kG}{u_*}(\cos\alpha - \sin\alpha), \quad (3)$$

so that $\alpha \rightarrow 45°$ as $h \rightarrow 0$. Continuity of shear stress implies

$$\sqrt{2Kf}\, G \sin\alpha = u_*^2 \quad (4)$$

while the continuity of eddy viscosity (as required for the separate continuity of shear) gives

$$K = ku_*(h + z_o), \quad (5)$$

with $K$ denoting the constant value within the Ekman layer. Essentially the same three matching conditions are presented on page 276 of the textbook by Haltiner and Williams (Ref. 2). Eqs. 3–5 constitute three equations in $z_o$, G, $K$, $u_*$, h, and $\alpha$, three of which must be specified for closure. For a general circulation model, the roughness and gradient wind may be regarded as externally specified parameters for the PBL. Given these, the fixed prescription of $K$ or h would provide a complete specification of the PBL. With $K = 10^3\ cm^2{\bullet}s^{-1}$, $z_o = 1$cm, G = 0.5 $m{\bullet}s^{-1}$, and $f = \Omega = 4.56 \times 10^{-6}\ s^{-1}$ on Titan, for example, the simultaneous solution of Eqs. 3–5 yields h = 22m, $\alpha = 21°$, and $u_* = 1.3\ cm{\bullet}s^{-1}$. In reality, the expected variation of both $K$ and h with the external parameters suggests the need either for a separate prediction of one of them or else some further scheme for the independent derivation of the surface stress. For the Titan PBL, in advance of any observations, the gradient wind may itself be regarded as essentially unknown. One approach to parametric closure is the separate specification of $K$, for example, as some empirically determined function of stability, as in the PBL scheme of the Model II version of the GISS General Circulation Model (Ref. 12). It is not clear, however, to what extent this parameterization would give reliable results for the different surface conditions on Titan. Before proceeding with a similarity approach to closure, some features of the stratified, diabatically forced structure of the PBL will be briefly reviewed.

## 3. OBUKHOV SCALING

The foregoing discussion strictly applies only to neutrally buoyant stability. Under diabatic conditions, a relevant scale for the turbulent surface layer is the Obukhov length (Ref.13), defined as $L \equiv -\rho c_p \theta_o u_*^3/kgQ_T$, where $c_p$ is the specific heat at constrant pressure, $\theta_o$ is the potential temperature, g is the gravitational acceleration, and $Q_T$ is the turbulent heat flux (defined as postive upwards). Although the precise value of $Q_T$ is unknown for the Titan boundary layer, and probably varies diurnally in sign as well as size, it is almost certainly always smaller in magnitude than the absorbed solar flux at normal incidence, $Q_S = 4\sigma T_e^4 \approx 12$ W•m$^{-2}$, and under global mean conditions is probably less than $Q_S/4 \approx 3$ W•m$^{-2}$. Golitsyn (Ref. 14) points out that in the terrestrial boundary layer, under daytime conditions of fully developed convection, the turbulent flux is typically less than one-tenth the solar constant, while the size of the downward turbulent flux at night is several times lower. Fig. 1 displays as solid lines the magnitude of the Obukhov scale for Titan (with $\rho c_p \theta_o/kg = 1.14 \times 10^6$ W•s$^3$m$^{-4}$) as a function of the friction velocity for a variety of turbulent heat flux values. For unstable conditions, |L| may be physically interpreted as roughly (three times) the maximum height z at which the friction velocity exceeds the vertical convective velocity $w' \sim (Q_T gz/\rho c_p \theta_o)^{1/3}$, as predicted by mixing length theory and confirmed (within a factor of two or three) by field observations (Ref. 15). The |L| vs. $u_*$ lines in Fig. 1 may therefore also be used to estimate the size of convective velocities within the Titan PBL. Although for stable conditions the convection is essentially forced by the wind shear, the Obukhov length again provides a measure of (roughly twice) the elevation at which the momentum mixing is limited by buoyancy (Ref.16).

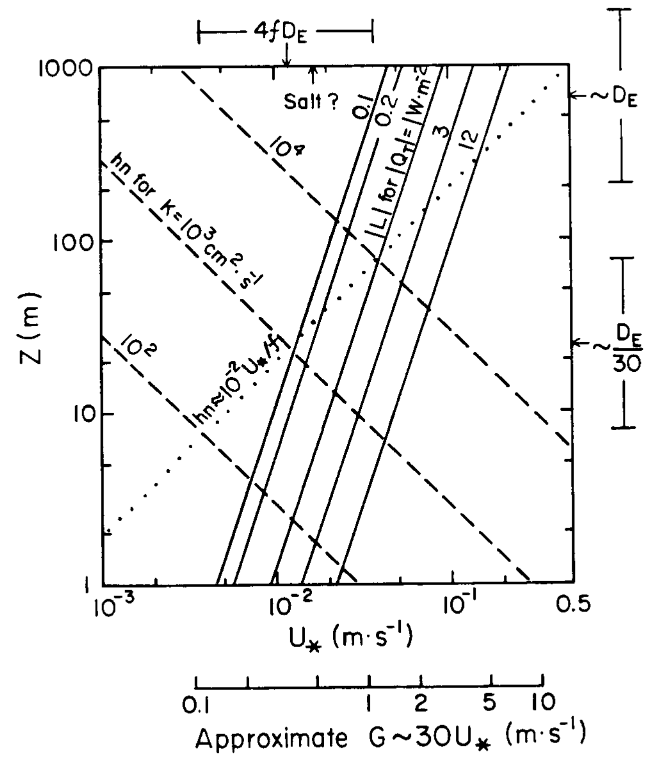


Figure 1. Height scales for the Titan PBL as a function of fricition velocity. Solid lines show the magnitude of the Obukhov length for the indicated fluxes, dashed lines the height of the neutral surface layer for a fixed choice of $K$, and the dotted line the surface layer height as constrained by similarity theory. Also indicated at the top is the similarity estimate of the friction velocity and along the right-side ordinate the Ekman depth $D_E$ and surface layer height for the range $K = 10^2 - 10^4$cm$^2$•s$^{-1}$.

Under diabatic conditions the vertical shear (and stability) profiles within the surface layer may be generalized by the Monin-Obukhov functions of z/L (Ref.17), with some attendant modification of the Ekman matching Eqs. 3 and 5 (Ref. 2). The eddy viscosity at the top of the surface layer, for example, is accordingly specified in terms of the evaluated function

$$\phi_m(h) = ku_*(h+z_o)/K = \begin{cases} 1+5\,(h+z_o)/L & \text{for } L>0 \\ [1-16(h+z_o)/L]^{-1/4} & \text{for } L<0. \end{cases} \quad (6)$$

The adopted Businger-Dyer formulas (Ref.18) for $\phi_m$ are empirically determined from micrometeorology measurements but can also be derived by mixing length arguments, with the coefficient "5" for the stable case corresponding to the inverse critical Richardson number for the onset of turbulence (Ref.19). The Obukhov similarity formalism emphasizes the fact that exactly netural stability, with $\phi_m$=1, is in some sense a singular case, for which $L \rightarrow \infty$ (or $h+z_o \rightarrow 0$), and is rarely observed. More typically, $h \sim O(|L|)$. Using Eq. 5 to define a netural surface layer height $h_n \equiv K/ku_*$ (having neglected the dfference between this and the relatively small roughness scale $z_o$), Eq. 6 may be used to estimate its relationship to L for the same values of $K$ and $u_*$. For moderate to strong stability (h/L > 0.5), $L \approx 5$–$7\ h_n$, while for moderate to strong instability (–h/L > 0.5), $L \leq h_n$, with $L \approx -0.1 h_n$ for –h/L = 3 (comparable to the extreme limit of terrestrial surface layer measurements). For comparison, Fig.1 displays $h_n \equiv K/ku^*$ for three fixed choices of $K$ as dashed lines (running from the upper left to the lower right). The inspection of the plotted relationships between $h_n$ and |L| for Titan reveals, for example, that with $K = 10^3$ cm$^2$•s$^{-1}$, $u_* \approx 10^{-2}$m•s$^{-1}$, and $-L \approx 0.1 h_n \approx 2$m, the turbulent flux would appoach 1 W•m$^{-2}$, about one-tenth of the available solar drive. While the apparent order-of-magnitude similarity of the Obukhov scale and the neutral surface layer height is diagnostic of the diabatic constraints upon the real PBL, it is probably insufficiently precise to serve as the first-order closure assumption for Eqs. 3–5.

## 4. ROSSBY SIMILARITY THEORY

As discussed by Csanady (Ref.20), the evaluation of the surface stress by the vertical integration of the momentum balance Eq.1, assuming the over-lapping validity of the inner and outer solutions with a functional dependence similar to that explicitly summarized by Eq.2, demands that the Ekman depth scale in constant proportion to $u_*/f$. Blackadaar and Tennekes (Ref.21) reach the same conclusion by the asymptotic analysis of the PBL energy budget. Consistency with the definitions $h_n \equiv K/ku^*$ and $D_E \equiv \pi(2K/f)^{1/2}$ implies that this is equivalent to the requirement that the ratio $D_E/h_n$ is also constant, with

$$r \equiv u_*/fD_E = (D_E/h_n)/2k\pi^2. \quad (7)$$

Using Eq. 7 and the definition of $D_E$, together with the Ekman matching conditions specified by Eqs. 3, 4, and 5 (again with $h \cong h+z_o=h_n$), it follows (after some algebraic manipulation) that

$$ln\frac{G}{fz_o} = A + ln\frac{G}{u_*} + [(\frac{kG}{u_*})^2 - B^2]^{1/2} \quad (8)$$

and

$$\sin\alpha = \frac{B}{k}(u_*/G) \quad (9)$$

where the constants A and B are given by this derivation as

$$A = ln(2r^2\pi^2k) - B; \text{ with } B = r\pi k. \quad (10)$$

Eq. 8 is the universal "geostrophic resistance law" (cf. Ref. 20) prescribing the relationship of the drag coefficient $C_D \equiv (u_*/G)^2$ to the surface gradient wind, surface roughness, and Coriolis parameter. The ratio $G/fz_0$ may be referred to as the surface Rossby number. Eq. 9 prescribes the cross-isobaric turning angle between the surface wind and the gradient flow aloft. Eqs. 8 and 9, referred to as the Rossby (or Kasinski–Monin) similarity relations, have been variously derived by dimensional analysis, asymptotic matching, and phenomenological arguments (e.g. Refs. 20, 21, 22) and are not limited to $K$ theory models, with A and B (sometimes reversed in the literature) typically regarded as independent empirical parameters.

As suggested by Blackadar and Tennekes (Ref.21), however, and explicitly developed by Brown (Ref.23), the derivations of A and B from the matched Ekman-surface layer model are fixed by the constant similarity ratio defined in Eq. 7. The presumed smallness of the surface layer as compared with the full depth of the PBL requires that $r \equiv u_*/fD_E$ is of order unity or larger. Assuming $D_E/h_n \geq 10$, for example, Eq. 7 specifies that $r \geq 1.4$. The actual ratio of $u_*/f$ to the PBL depth has been variously estimated from terrestrial field data for near-neutral conditions as between 2.9 and 4.0 (Refs.21,24, 25). Taking r=4, Eq.7 gives $A \cong 0.31$ with $B \cong 4.4$. For comparison, the analysis of data from the Wangara (Australia) field experiment by Clarke and Hess (Ref.26) indicates that $A = 1.1 \pm 0.5$ and $B = 4.3 \pm 0.7$ under neutral conditions. Fig.2 displays the resistance law of Eq.8 for neutral stability using the theoretically derived values of A and B corresponding to r=4, also marked with representative values of the turn-angle prescribed by Eq.9. The three inset scales indicate the surface gradient wind velocity on Titan corresponding to the universal surface Rossby number for latitude 30° and three different choices of the roughness parameter.

Under diabatic conditions the parameters A and B are presumably a function of stability. In principle, the generalization of Eqs. 3 and 5 as prescribed by the Monin-Obukhov similarity functions could again be used to derive these from the Ekman-surface model (cf.Ref.23). Field measurements suggest that the similarity ratio for the PBL depth may itself vary with stability, at least for $|D_E/L| > 200$ (Ref.25), so that some further assumption may be required to extend this procedure to arbitrary stabilities. For the extreme limit of unstable convection (with $-L \rightarrow 0$), however, Eq.6 implies that $K \rightarrow \infty$. The comparison of Eqs.4 and 9 then suggests that in this case the turn angle and therefore B must asymptotically vanish. This expectation is corroborated by the field measurements (Ref.26), which suggest that for $ku_*/fL < -100$, $A \cong 5$ and $B \cong 0$. (It is probably only fortuitous that Eq.10, again with r=4 but modified for B=0, gives A=4.7, since the argument within the logarithm for the given expression must also change with B.) The resulting drag law is indicated by the dashed curve in the upper portion of Fig.2. For stable condtions, both the field data (Ref.26) and the similarity calculation by Brown (Ref.23) suggest that A is negative, decreasing for smaller L, while B is positive and increasing. For illustration, the resistance law for $A \cong -4$ and $B \cong 9$ is plotted as the lower dashed curve in Fig.2, corresponding to moderately stable conditions with $ku_*/fL \approx 30$, for which $L \sim D_E/30 \sim h$. Within the parameter range of interest to Titan, Fig.2 shows that $(u_*/G)$ probably falls between 0.015 for stable conditions and 0.05 for fully developed convection, with $C_D \equiv (u_*/G)^2 \cong 0.2\text{–}3 \times 10^{-3}$. For weakly convective conditions, with $z_0 \approx 1\text{–}10$ cm, $G \approx 30u_*$ should be a good estimate of the drag law and has been used to calibrate the auxillary scale for the surface wind given at the bottom of Fig.1.

These conclusions as to the strength of the surface drag have been derived from the Rossby similarity theory without any reliance upon the specific value of the eddy viscosity. Assuming the similarity ratio expressed by Eq. 7 is maintained for moderately diabatic conditions,with $|D_E/L|<100$ (cf.Refs.23, 25), it may be used to place an order-of-magnitude constrait on $K$, by comparison of the PBL scale depths illustrated in Fig.1. With

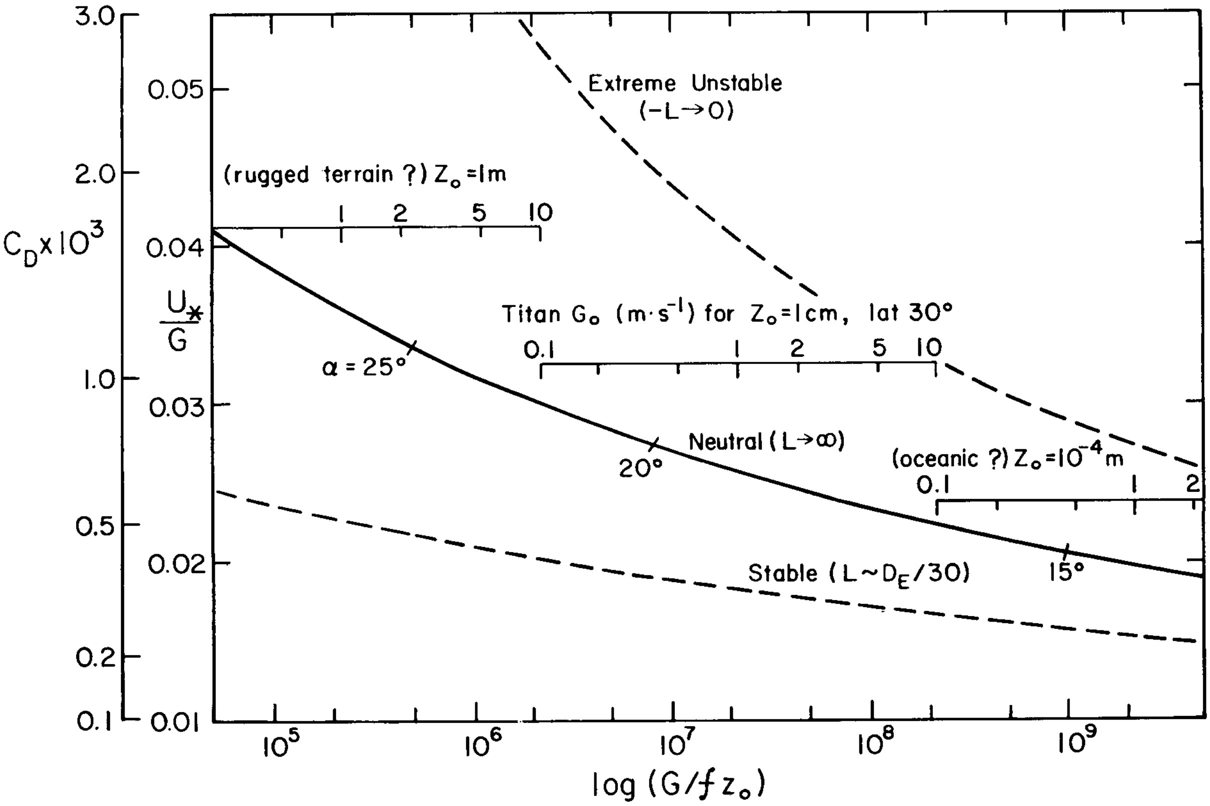


Fig.2. The univsersal resistance law for surface gradient flow as a function of the surface Rossby number $G/fz_0$ under neutral, extreme unstable (convective), and moderately stable conditions. The inset scales indicate the corresponding surface gradient wind velocity at latitude 30° on Titan, with three different choices for the surface roughness.

$r = 4$, Eq.7 implies that $D_E/h_n = 2k\pi^2 r \cong 28$ and $u^*/fh_n = 2k(r\pi)^2 \cong 111$. The inferred relationship between the neutral surface layer height and the friction velocity is plotted as the dotted line extending from the lower left to the upper right of Fig.1. Assuming the height of the neutrally stratified surface layer as given by the similarity theory is comparable to the Obukhov length, i.e. $h_n \approx 10^{-2}u^*/f \sim O(|L|)$, with a turbulent heat flux no larger than about 10% of the global mean solar absorption, the intersection of the dotted line with the corresponding solid line for $|L|$ corresponds to an upper limit on the eddy viscosity ~$10^4$ $cm^2 \cdot s^{-1}$. For a larger $K$, the turbulent flux would exceed the assumed limit. To the extent that the actual flux is smaller or $-L < h_n$ (as for vigorous convection) the implied limit on $K$ would itself be smaller. This argument represents an order-of-magnitude corroboration of the estimated limit for $K$ on Titan by Flasar *et al.* (Ref.6).

## 5. SURFACE SPEED LIMITS

Assuming a fixed upper value for the eddy mixing coefficient $K_{max}$, the derived resistance law may be used to estimate an upper limit on the strength of the surface gradient wind. Using the definition of the Ekman depth and drag coefficient, together with the similarity assumption (Eq.7), with $r = 4$, the implied surface speed limit is

$$G_{max} = 4\pi(2fK_{max}/C_D)^{1/2} \tag{11}$$

Since $C_D$ itself varies somewhat with G, as well as the choice of f and $z_0$, a precise estimate requires the iterative solution of Eq.11 with Eq.8. With $z_0$=1cm, $f(30°) = \Omega$, and $K_{max} = 10^3 cm^2 \cdot s^{-1}$ the iterated solution for neutral stability gives $G_{max} = 0.44$ $m \cdot s^{-1}$ or $G_{max} = 1.5$ $m \cdot s^{-1}$ for $K_{max} = 10^4 cm^2 \cdot s^{-1}$. In rugged terrain, with $z_0 = 1$m, the speed limits are smaller, while for $z_0 = 10^{-4}$ m, as appropriate for the relatively smooth conditions of a lake or sea, the limits become $G_{max} = 0.60$ and 2.0 $m \cdot s^{-1}$ for $K_{max} = 10^3$ and $10^4$ $cm^2 \cdot s^{-1}$, respectively. These limits could be slightly larger under strongly stable conditions, but probably by less than a factor of two.

## 6. SALTATION

The estimation of the surface stress is relevant to the possible aerodynamic lifting of dust into Titan's atmosphere. The threshold friction velocity for windblown grain motion ("saltation") is empirically given by $u_*^{salt} = A_B(g\rho_p D_p/\rho)^{1/2}$ (Ref.27) where g is the gravitational acceleration, $\rho_p$ and $D_p$ are the particle density and diameter, $\rho$ is the density of the atmosphere and the Bagnold parameter $A_B$ is a function of the particle friction Reynolds number, $Re_* \equiv u_* D_p/\upsilon$, with $\upsilon$ the kinematic *molecular* viscosity. Observations suggest that for $Re_*>1$, $A_B \approx 0.1$, but increases sharply for $Re_*$ smaller than roughly 0.5, depending somewhat on the strength of interparticle forces, the density ratio $\rho_p/\rho$, and the surface roughness (Ref.28). There is therefore a minimum threshold friction speed associated with an optimal particle radius for which slatation can occur. The saltation formula may be derived from Stoke's law for the viscous drag force on a sphere, with the Bagnold parameter given in terms of the aerodynamic Reynolds number Re as $A_B = (Re/18)^{1/2}$. Identifying Re in this expression with $Re_* \equiv u_* D_p/\upsilon = 0.5$, $D_p$ may be eliminated from the saltation formula to estimate the minimum friction velocity as

$$u_*^{salt} \approx 0.24\,(\upsilon g \rho_p/\rho)^{1/3} \tag{12}$$

with

$$D_p^{opt} \approx 0.5\upsilon/u_*^{salt} \tag{13}$$

With the appropriate choice of parameters for Mars, Eq.12 is in good agreement with applicable wind tunnel determinations (Ref.28) of the threshold friction velocity, while Eq.13 oversestimates the optimal particle diamter by around a factor of 2. Estimating the molecular viscosity for Titan by temperature-density scaling with respect to measured values for the Earth's atmosphere, $\upsilon \approx 0.146$ $cm^2 \cdot s^{-1}(95K/293K)^{1/2}(1.2/5.5) \approx 0.018$ $cm^2 \cdot s^{-1}$. Also with $g = 1.35$ $m \cdot s^{-2}$, $\rho \approx 5.5 \times 10^{-3} g \cdot cm^{-3}$, and adopting $\rho_p \approx 0.75$ $g \cdot cm^{-3}$, as appropriate for some mixture of ices and condensed organics (Ref.29), Eqs. 12 and 13 give $u_*^{salt} \approx 1.7$ $cm \cdot s^{-1}$ and $D_p^{opt} \approx 50\mu m$. Since the estimated threshold only slightly exceeds the nominal estimate of the friction velocity, it is possible that windblown dust is an occasional feature of the Titan meteorology, depending upon the availability of sub-millimeter sized grains at its surface.

## 7. SUMMARY

Table 2 summarizes the major results of the preliminary assessment of the Titan planetary boundary layer. These should provide a guide to the parameterization of surface drag appropriate to realistic models and numerical simulations of Titan's general circulation. It would be of interest to investigate, for example, whether the relatively long spin-down time plays a role in the selection of quasi-barotropic eddies as a dominant feature of the Titan atmospheric dynamics (cf.Ref.30).

The PBL estimates will be tested *in situ* during the last two or three minutes of the atmospheric descent of the Huygens Titan Probe, with measurements of velocity and temperature by the Huygens Atmospheric Structure Instrument, the Descent Imager Spectral Radiometer, and the Doppler Wind Experiment. It is possible, for example, that the image tracking of surface features, together with the Doppler measurements, will provide an estimate not only of the near-surface velocity but also its rotation with altitude, corresponding to the cross-isobaric turn angle. It may also be possible to characterize the surface roughness parameter by Cassini Orbiter RADAR measurements (cf.Ref.31).

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Table 2. The Titan PBL – An Executive Summary.

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|---|---|---|
| Eddy viscosity | $K$ | ~$10^3$ $cm^2 s^{-1}$ |
| Ekman PBL depth | $D_E$ | ~0.7 km |
| Surface layer height | h | ~20 m |
| Friction velocity | $u_*$ | ~1 $cm \cdot s^{-1}$ |
| Saltation threshold | $u_*^S$ | ≈1.6 $cm \cdot s^{-1}$ |
| Geostrophic drag coefficient | $C_D \equiv (u^*/G)^2$ | ~$10^{-3}$ |
| Spin-down time | $\tau_E$ | ≈ 500 days |
| Surface gradient wind | G | < 2 $m \cdot s^{-1}$ |
| Turn angle | $\alpha$ | ≈ 20° |

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